A STERILE NEUTRINO SCENARIO CONSTRAINED BY EXPERIMENTS AND COSMOLOGY

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Abstract

We discuss a model in which three active and one sterile neutrino account for the solar, the atmospheric and the LSND neutrino anomalies. It is shown that if $N_\nu < 4$ then these and other experiments and big bang nucleosynthesis constrain all the mixing angles severely, and allow only the small-angle MSW solution. If these neutrinos are of Majorana type, then negative results of neutrinoless double beta decay experiments imply that the total mass of neutrinos is not sufficient to account for all the hot dark matter components.

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1 Introduction

Recently LSND group [1] (See also [2]) reported that they have found candi-
date events for $\overline{\nu}_\mu \to \overline{\nu}_e$ oscillation. Combing their result in [1] with the data
by the E776 group [3] on the same channel $\nu_\mu \to \nu_e$ and the reactor data by
Bugey [4], the mass squared difference Δm^2 seems to be in the range

$$0.27\text{eV}^2 \lesssim \Delta m^2_{\text{LSND}} \lesssim 2.3\text{eV}^2. \tag{1}$$

On the other hand, it has been argued that the solar neutrino problem [5]–
[8] is solved if a set of the oscillation parameters of the two-flavor neutrino
mixing satisfies one of the followings [9]–[14]:

$$
\begin{aligned}
&(\Delta m^2_\odot, \sin^2 2\theta_\odot) \\
&\simeq \left\{
\begin{array}{ll}
(\mathcal{O}(10^{-10}\text{eV}^2), \mathcal{O}(1)), & (\text{vacuum oscillation solution}) \\
(\mathcal{O}(10^{-5}\text{eV}^2), \mathcal{O}(10^{-2})), & (\text{small} - \text{angle MSW solution}) \\
(\mathcal{O}(10^{-5}\text{eV}^2), \mathcal{O}(1)) & (\text{large} - \text{angle MSW solution}).
\end{array}
\right.
\end{aligned}
\tag{2}
$$

Furthermore, it has been reported by the Kamiokande group [15][16] that
their atmospheric neutrino data suggests neutrino oscillation with a set of
parameters $(\Delta m^2, \ \sin^2 2\theta) \simeq (1.8 \times 10^{-2}\text{eV}^2, 1.0)$ for $(\nu_e \leftrightarrow \nu_\mu)$, $(1.6 \times 10^{-2}\text{eV}^2, 1.0)$ for $(\nu_\mu \leftrightarrow \nu_\tau)$ (See also [17]–[20]).

The results of experiments on these anomalies have been given in the
framework of two-flavor mixings in the original literatures, and have been
analyzed from the viewpoint of the three flavor mixing by many people [21]-
[26]. It has been shown that strong constraints are obtained if two of the
three anomalies are taken for granted, and it seems difficult [27] (See also
[28]) to account for all these three anomalies within the three flavor mixing.

Here we investigate the possibility in which one sterile neutrino as well as
three known flavors of neutrinos are responsible for these anomalies. Some
features of this possibility have been discussed in the past [29]–[34]. In this

paper we analyze in detail the mass squared differences and the mixing angles in this scenario. It turns out that if the number N_ν of light neutrinos is less than 4, then because of strong constraints by the solar and atmospheric neutrino observations, accelerator and reactor experiments including LSND as well as big bang nucleosynthesis, all the 6 mixing angles are strongly constrained. In this case 4×4 mixing matrix is effectively split into 2×2 $\nu_e \leftrightarrow \nu_s$ and $\nu_\mu \leftrightarrow \nu_\tau$ matrices, and only the small-angle MSW solution is allowed. If we assume that these neutrinos are of Majorana type, then the upper bound on $\langle m_{\nu_e} \rangle$ from neutrinoless double β decay experiments suggests that these neutrinos are not heavy enough to explain all the hot dark matter components. If $N_\nu \geq 4$, then all the solutions to the solar neutrino problem are allowed, hot dark matter can be accounted for by neutrinos and there may be a chance to observe neutrinoless double β decays in the future experiments.

It has been pointed out that physics of supernova gives constraints on sterile neutrinos [35][36], but these constraints apply only to Δm^2 which is larger than the mass scale suggested by the LSND data, so we will not discuss this point in this paper.

In section 2 we present our formalism on oscillations among four species of neutrinos. In section 3 we discuss constraints from reactor and accelerator experiments. In section 4 constraints by big bang nucleosynthesis are considered. In section 5 we give constraints from neutrinoless double β decay experiments. In section 6 we examine the possibility that neutrinos are hot dark matter. In section 7 we discuss consequences in case of $N_\nu \geq 4$. In section 8 we give our conclusions.

2 Oscillations Among Four Flavors

Let us consider a model with three flavors of neutrinos ν_e, ν_μ, ν_τ and one sterile neutrino ν_s, which is singlet with respect to all the gauge groups in the standard model. We assume that $(\nu_e,\ \nu_\mu,\ \nu_\tau,\ \nu_s)$ are related to the mass eigenstates $(\nu_1,\ \nu_2,\ \nu_3,\ \nu_4)$ by the following unitary matrix:

$$
\begin{aligned}
U & \equiv
\begin{pmatrix}
U_{e1} & U_{e2} & U_{e3} & U_{e4} \\
U_{\mu 1} & U_{\mu 2} & U_{\mu 3} & U_{\mu 4} \\
U_{\tau 1} & U_{\tau 2} & U_{\tau 3} & U_{\tau 4} \\
U_{s1} & U_{s2} & U_{s3} & U_{s4}
\end{pmatrix} \\
& \equiv e^{i\alpha'}e^{i\beta'\lambda_3}e^{\sqrt{3}i\gamma'\lambda_8}e^{\sqrt{6}i\delta'\lambda_{15}}V_{KM}e^{-\sqrt{6}i\gamma\lambda_{15}}e^{-\sqrt{3}i\beta\lambda_8}e^{-i\alpha\lambda_3}.
\end{aligned}
\tag{3}
$$

Here

$$
\begin{aligned}
V_{KM} & \equiv R_{34}(\frac{\pi}{2}-\theta_{34})R_{24}(\theta_{24})e^{i\delta_1\lambda_3}R_{23}(\frac{\pi}{2}-\theta_{23})e^{-i\delta_1\lambda_3} \\
& \quad e^{i\delta_3\lambda_{15}}R_{14}(\theta_{14})e^{-i\delta_3\lambda_{15}}e^{i\delta_2\lambda_8}R_{13}(\theta_{13})e^{-i\delta_2\lambda_8}R_{12}(\theta_{12})
\end{aligned}
\tag{4}
$$

is a 4×4 Kobayashi-Maskawa matrix for the lepton sector, $c_{ij}\equiv\cos\theta_{ij}, s_{ij}\equiv\sin\theta_{ij}$,

$$
R_{jk}(\theta) \equiv \exp\left(iT_{jk}\theta\right)
\tag{5}
$$

is a 4×4 orthogonal matrix with

$$
(T_{jk})_{\ell m} = i\left(\delta_{j\ell}\delta_{km}-\delta_{jm}\delta_{k\ell}\right),
\tag{6}
$$

$2\lambda_3 \equiv \mathrm{diag}(1,-1,0,0)$, $2\sqrt{3}\lambda_8 \equiv \mathrm{diag}(1,1,2,0)$, $2\sqrt{6}\lambda_{15} \equiv \mathrm{diag}(1,1,1,-3)$ are diagonal elements of the $su(4)$ generators.

The four phases α', β', γ', δ' in front of V_{KM} in (3) can be absorbed by redefining the wave functions of charged leptons. If there is a Majorana mass term, however, the three phases α, β, γ cannot be absorbed [37]–[41]. Since these factors α, β, γ are cancelled in the probability of neutrino oscillations,

they do not affect the results of neutrino oscillations, but as we will see later, they do affect the effective mass of ν_e and the constraint on the masses of neutrinos from neutrinoless double β decay.

We can assume without loss of generality that

$$m_1^2 < m_2^2 < m_3^2 < m_4^2. \tag{7}$$

Three mass scales $\Delta m_\odot^2 \sim \mathcal{O}(10^{-5}\mathrm{eV}^2)$ or $\mathcal{O}(10^{-10}\mathrm{eV}^2)$, $\Delta m_{\mathrm{atm}}^2 \sim \mathcal{O}(10^{-2}\mathrm{eV}^2)$, $\Delta m_{\mathrm{LSND}}^2 \sim \mathcal{O}(1\mathrm{eV}^2)$ seem to be necessary to explain the suppression of the ^{7}Be solar neutrinos [9], the zenith angle dependence of the Kamiokande multi-GeV data of atmospheric neutrinos [16], and the LSND data [1],[4] so we will look for solutions of neutrino oscillations among four species where the mass squared differences are the three mass scales mentioned above.

In the present case there are six possibilities, and they are classified into two categories. One is a case in which two mass eigenstates have different degenerate mass scales:

(ia)

$$m_1^2 \simeq m_2^2 \quad \ll \quad m_3^2 \simeq m_4^2$$
$$\text{with} \quad (\Delta m_{21}^2, \Delta m_{43}^2) = (\Delta m_\odot^2, \Delta m_{\mathrm{atm}}^2)$$
$$\Delta m_{jk}^2 = \Delta m_{\mathrm{LSND}}^2 \text{ for } j = 3, 4, k = 1, 2, \tag{8}$$

(ib)

$$m_1^2 \simeq m_2^2 \quad \ll \quad m_3^2 \simeq m_4^2$$
$$\text{with} \quad (\Delta m_{21}^2, \Delta m_{43}^2) = (\Delta m_{\mathrm{atm}}^2, \Delta m_\odot^2)$$
$$\Delta m_{jk}^2 = \Delta m_{\mathrm{LSND}}^2 \text{ for } j = 3, 4, k = 1, 2. \tag{9}$$

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[4]It can be shown that the probability of neutrino oscillation in vacuum in the framework of more than three flavors is reduced essentially to the two flavor case, once one assumes the mass hierarchy [23][24], so that the analysis of [1] indicates that the mass squared difference in our case should satisfy (1).

Another is a case in which three mass eigenstates have degenerate masses while one eigenstate has a mass far from others:

(iia)

$$m_1^2 \simeq m_2^2 \simeq m_3^2 \quad \ll \quad m_4^2$$
$$\text{with} \quad (\Delta m_{21}^2, \Delta m_{32}^2) = (\Delta m_\odot^2, \Delta m_{\text{atm}}^2)$$
$$\Delta m_{4j}^2 = \Delta m_{\text{LSND}}^2 \text{ for } j = 1, 2, 3, \quad (10)$$

(iib)

$$m_1^2 \simeq m_2^2 \simeq m_3^2 \quad \ll \quad m_4^2$$
$$\text{with} \quad (\Delta m_{21}^2, \Delta m_{32}^2) = (\Delta m_{\text{atm}}^2, \Delta m_\odot^2)$$
$$\Delta m_{4j}^2 = \Delta m_{\text{LSND}}^2 \text{ for } j = 1, 2, 3, \quad (11)$$

(iic)

$$m_1^2 \quad \ll \quad m_2^2 \simeq m_3^2 \simeq m_4^2$$
$$\text{with} \quad (\Delta m_{32}^2, \Delta m_{43}^2) = (\Delta m_\odot^2, \Delta m_{\text{atm}}^2)$$
$$\Delta m_{j1}^2 = \Delta m_{\text{LSND}}^2 \text{ for } j = 2, 3, 4, \quad (12)$$

(iid)

$$m_1^2 \quad \ll \quad m_2^2 \simeq m_3^2 \simeq m_4^2$$
$$\text{with} \quad (\Delta m_{32}^2, \Delta m_{43}^2) = (\Delta m_{\text{atm}}^2, \Delta m_\odot^2)$$
$$\Delta m_{j1}^2 = \Delta m_{\text{LSND}}^2 \text{ for } j = 2, 3, 4. \quad (13)$$

The latter four possibilities (ii-a) – (ii-d) are excluded by reactor and accelerator experiments, as we will show below. The case (ib) can be treated in exactly the same manner as (ia) by changing the labels 1↔3, 2↔4 of the mass eigenstates, so in sections 3 and 4 we will discuss only the case (ia) for simplicity.

3 Reactor and Accelerator experiments

Let us discuss constraints from reactor and accelerator experiments. The probability $P(\nu_\alpha \to \nu_\beta)$ of the transition $\nu_\alpha \to \nu_\beta$ in vacuum is given by

$$P(\nu_\alpha \to \nu_\beta) = -4 \sum_{i<j} U_{\alpha i} U_{\alpha j}^* U_{\beta i}^* U_{\beta j} \sin^2 \left(\frac{\Delta E_{ij} L}{2} \right)$$
$$-2 \sum_{i<j} \text{Im} \left(U_{\alpha i} U_{\alpha j}^* U_{\beta i}^* U_{\beta j} \right) \sin \left(\Delta E_{ij} L \right) \qquad (14)$$

$$P(\nu_\alpha \to \nu_\alpha) = 1 - 4 \sum_{i<j} |U_{\alpha i}|^2 |U_{\alpha j}|^2 \sin^2 \left(\frac{\Delta E_{ij} L}{2} \right), \qquad (15)$$

where $\alpha = e, \mu, \tau, s$ are the flavor indices, $\Delta E_{ij} \equiv E_i - E_j = \sqrt{p^2 + m_i^2} - \sqrt{p^2 + m_j^2} \simeq (m_i^2 - m_j^2)/2E \equiv \Delta m_{ij}^2/2E$ is the difference of the energy of the two mass eigenstates. (14) and (15) are exact expressions in vacuum, and they are simplified if we assume the patterns (8) – (13) with mass hierarchy.

3.1 Mass Pattern (i)

Applying the formula (14) and (15) to the present case (8) with mass hierarchy, we obtain the following expressions:

$$P(\nu_\alpha \to \nu_\beta) \simeq 4 \left| \sum_{j=3}^{4} U_{\alpha j} U_{\beta j}^* \right|^2 \sin^2 \left(\frac{\Delta E_{31} L}{2} \right) \qquad (16)$$

$$P(\nu_\alpha \to \nu_\alpha) \simeq 1 - 4(|U_{\alpha 1}|^2 + |U_{\alpha 2}|^2)(|U_{\alpha 3}|^2 + |U_{\alpha 4}|^2) \sin^2 \left(\frac{\Delta E_{31} L}{2} \right) \qquad (17)$$

where the CP violating terms have been dropped out in (16). From (17) the negative result of a disappearance experiment measured at distances L_1 and L_2 gives a condition:

$$\epsilon > \quad 4|U_{\alpha 1}|^2 |U_{\alpha 2}|^2 \epsilon f_\alpha(\Delta m_{21}^2)$$
$$+ \quad 4(|U_{\alpha 1}|^2 + |U_{\alpha 2}|^2)(|U_{\alpha 3}|^2 + |U_{\alpha 4}|^2) \epsilon f_\alpha(\Delta m_{31}^2)$$
$$+ \quad 4|U_{\alpha 3}|^2 |U_{\alpha 4}|^2 \epsilon f_\alpha(\Delta m_{43}^2). \qquad (18)$$

$f_\alpha(\Delta m^2)$ in (18) is defined by

$$
\begin{aligned}
f_\alpha(\Delta m^2) \;\equiv\; & \frac{1}{\epsilon}\left[\left\langle \sin^2\left(\frac{\Delta m^2 L_1}{4E}\right)\right\rangle_{\alpha\to\alpha} - \left\langle \sin^2\left(\frac{\Delta m^2 L_2}{4E}\right)\right\rangle_{\alpha\to\alpha}\right] \\
& \begin{cases} = 1/\sin^2 2\theta_\alpha(\Delta m^2) & \text{for } f_\alpha(\Delta m^2) \geq 1 \\ \simeq \text{const.}(\Delta m^2)^2 & \text{for } \left\langle \sin^2\left(\frac{\Delta m^2 L_1}{4E}\right)\right\rangle \ll 1, \end{cases}
\end{aligned}
\tag{19}
$$

where

$$
\begin{aligned}
& \left\langle \sin^2\left(\frac{\Delta m^2 L}{4E}\right)\right\rangle_{\alpha\to\beta} \\
\equiv\; & \frac{1}{N_{\alpha\beta}(L)} n_T \int_0^\infty dE \int_0^{q_{\max}} dq\; \epsilon(q) F_\alpha(E) \frac{d\sigma_\beta(E,q)}{dq} \sin^2\left(\frac{\Delta m^2 L}{4E}\right),
\end{aligned}
\tag{20}
$$

$$
N_{\alpha\beta}(L) \;\equiv\; n_T \int_0^\infty dE \int_0^{q_{\max}} dq\; \epsilon(q) F_\alpha(E) \frac{d\sigma_\beta(E,q)}{dq},
\tag{21}
$$

$F_\alpha(E)$ is the flux of neutrino ν_α with energy E, n_T is the number of target nucleons, $\epsilon(q)$ is the detection efficiency function for charged leptons ℓ_β of energy q, $d\sigma_\beta(E,q)/dq$ is the differential cross section of the interaction $\nu_\beta X \to \ell_\beta X'$, and ϵ stands for the largest fraction of the appearance events allowed by a given confidence level.

Let us first discuss the reactor experiment [4]. In this case we have $|\Delta m_{21}^2 L/4E| \ll 1$, so that (18) becomes

$$
\begin{aligned}
\frac{1}{4} \;>\; & (|U_{e3}|^2 + |U_{e4}|^2)(1 - |U_{e3}|^2 - |U_{e4}|^2) f_{\text{Bugey}}\left(\Delta m_{31}^2\right) \\
& + |U_{e3}|^2 |U_{e4}|^2 f_{\text{Bugey}}\left(\Delta m_{43}^2\right),
\end{aligned}
\tag{22}
$$

where $f_{\text{Bugey}}\left(\Delta m_{31}^2\right)$ stands for $f_e\left(\Delta m_{31}^2\right)$ in (19) in case of the Bugey experiment [4]. From the $(\Delta m^2, \sin^2 2\theta)$ plot in [4] we have

$$
\begin{aligned}
6 \times 10^{-2} \lesssim\; & f_{\text{Bugey}}\left(\Delta m_{43}^2\right) \;\lesssim 25 \quad \text{for} \quad 3 \times 10^{-3}\text{eV}^2 \lesssim \Delta m_{43}^2 \lesssim 9 \times 10^{-2}\text{eV}^2 \\
11 \lesssim\; & f_{\text{Bugey}}\left(\Delta m_{31}^2\right) \;\lesssim 25 \quad \text{for} \quad 0.4\text{eV}^2 \lesssim \Delta m_{31}^2 \lesssim 2.5\text{eV}^2.
\end{aligned}
\tag{23}
$$

Since each term in (22) is semipositive definite, from (22) and (23) we get

$$|U_{e3}|^2 + |U_{e4}|^2 < \frac{1}{2}\left[1 - \sqrt{1 - 1/f_{\text{Bugey}}\left(\Delta m_{31}^2\right)}\right] \qquad (24)$$

or

$$|U_{e3}|^2 + |U_{e4}|^2 > \frac{1}{2}\left[1 + \sqrt{1 - 1/f_{\text{Bugey}}\left(\Delta m_{31}^2\right)}\right]. \qquad (25)$$

On the other hand, we have the following relation between the probabilities $P^{(2)}\left(\nu_e \to \nu_e; A(x)\right)$ and $P^{(4)}\left(\nu_e \to \nu_e; A(x)\right)$ for the solar neutrinos transitions in the two and four flavor mixings, respectively:

$$\begin{aligned} P^{(4)}\left(\nu_e \to \nu_e; A(x)\right) &= P^{(2)}\left(\nu_e \to \nu_e; (1 - |U_{e3}|^2 - |U_{e4}|^2)A(x)\right) \\ &\times \left(1 - |U_{e3}|^2 - |U_{e4}|^2\right)^2 + |U_{e3}|^2 + |U_{e4}|^2, \qquad (26) \end{aligned}$$

where $A(x)$ stands for the electron density, and we have averaged over rapid oscillations $\sin^2(\Delta m_{ij}^2 L/4E)$ for all $(i,j) \neq (2,1)$. Eq. (26) can be derived in a way similar to the case of the relation between $P^{(2)}$ and $P^{(3)}$ [42][43]. To account for the suppression of the ^{7}Be neutrino among all the solar neutrinos, $P^{(4)}\left(\nu_e \to \nu_e; A(x)\right)$ cannot be larger than $\frac{1}{2}$ [24], so the second possibility (25) is excluded. Using the relations $U_{e3} = s_{13}c_{14}e^{i\delta_2}$, $U_{e4} = s_{14}e^{i\delta_3}$, which can be derived from the expression (4), we have

$$s_{13}^2,\ s_{14}^2 \lesssim 1/f_{\text{Bugey}}\left(\Delta m_{31}^2\right) \lesssim \frac{1}{40} \quad \text{for} \quad 0.24\text{eV}^2 \lesssim \Delta m_{31}^2 \lesssim 2.5\text{eV}^2 \qquad (27)$$

or

$$\theta_{13},\ \theta_{14} \lesssim 9^\circ. \qquad (28)$$

Next let us consider the disappearance experiment of $\nu_\mu \to \nu_\mu$ [44]. In this case we have $|\Delta m_{21}^2 L/4E|, |\Delta m_{43}^2 L/4E| \ll 1$, so that

$$\frac{1}{4} > (|U_{\mu3}|^2 + |U_{\mu4}|^2)(1 - |U_{\mu3}|^2 - |U_{\mu4}|^2)\, f_{\text{CDHSW}}\left(\Delta m_{31}^2\right) \qquad (29)$$

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where $f_{\text{CDHSW}}\left(\Delta m_{31}^2\right)$ stands for $f_\mu\left(\Delta m_{31}^2\right)$ in (19) for the CDHSW data [44]. The allowed value for $f_{\text{CDHSW}}\left(\Delta m_{31}^2\right)$ is given by

$$1 \lesssim f_{\text{CDHSW}}\left(\Delta m_{31}^2\right) \lesssim 50 \quad \text{for} \quad 0.24\text{eV}^2 \le \Delta m_{31}^2 \le 2.5\text{eV}^2, \qquad (30)$$

and therefore we obtain

$$|U_{\mu 3}|^2 + |U_{\mu 4}|^2 < \frac{1}{2}\left[1 - \sqrt{1 - 1/f_{\text{CDHSW}}\left(\Delta m_{31}^2\right)}\right] \qquad (31)$$

or

$$|U_{\mu 3}|^2 + |U_{\mu 4}|^2 > \frac{1}{2}\left[1 + \sqrt{1 - 1/f_{\text{CDHSW}}\left(\Delta m_{31}^2\right)}\right]. \qquad (32)$$

Furthermore, we have the constraints from the atmospheric neutrino data. If we consider the probability $P(\nu_\mu \to \nu_\mu)$ for the multi-GeV data by the Kamiokande group, then we get

$$\begin{aligned}
P(\nu_\mu \to \nu_\mu) &= 1 - 4|U_{\mu 3}|^2|U_{\mu 4}|^2 \sin^2\left(\frac{\Delta m_{43}^2 L}{4E}\right) \\
&\quad - 2(|U_{\mu 3}|^2 + |U_{\mu 4}|^2)(1 - |U_{\mu 3}|^2 - |U_{\mu 4}|^2), \qquad (33)
\end{aligned}$$

where we have used $|\Delta m_{21}^2 L/4E| \ll 1$. To have the zenith angle dependence of the multi-GeV data, it is necessary for $P(\nu_\mu \to \nu_\mu)$ to deviate from unity to some extent. Using the constraints $s_{13}^2, s_{14}^2 \lesssim 1/40$ in (28) and the relations

$$\begin{aligned}
U_{\mu 1} &= -c_{12}c_{23}c_{24}s_{13}e^{-i\delta_2} + c_{24}s_{12}s_{23}e^{i\delta_1} - c_{12}c_{13}s_{14}s_{24}e^{-i\delta_3} \qquad (34) \\
U_{\mu 2} &= -c_{23}c_{24}s_{12}s_{13}e^{-i\delta_2} - c_{12}c_{24}s_{23}e^{i\delta_1} - c_{13}s_{12}s_{14}s_{24}e^{-i\delta_3}, \qquad (35)
\end{aligned}$$

we have

$$\begin{aligned}
|U_{\mu 1}|^2 + |U_{\mu 2}|^2 &\simeq s_{23}^2 c_{24}^2 \\
&< \frac{1}{2}\left[1 - \sqrt{1 - 1/f_{\text{CDHSW}}\left(\Delta m_{31}^2\right)}\right]. \qquad (36)
\end{aligned}$$

Because of the condition (28), the probability $P(\nu_e \to \nu_e)$ is close to unity for the mass squared difference $\Delta m^2 \sim \Delta m^2_{\text{atm}}$, so the only possible source for the atmospheric neutrino anomaly in our scenario is deviation of the probability $P(\nu_\mu \to \nu_\mu)$ from unity. Using the technique in [25], we find that the region of the parameters $|U_{\mu 3}|^2$ and $|U_{\mu 4}|^2$ allowed for the multi-GeV atmospheric neutrinos data of Kamiokande at 90% confidence level is

$$0 \le \theta_{23} \lesssim 50°, \ 25° \lesssim \theta_{24} \lesssim 55°, \tag{37}$$

where we have used the relations

$$U_{\mu 3} = c_{13}c_{23}c_{24} - s_{13}s_{14}s_{24}e^{i(\delta_2 - \delta_3)} \tag{38}$$
$$U_{\mu 4} = c_{14}s_{24}. \tag{39}$$

From (37) we have $s^2_{23}c^2_{24} \lesssim 0.48$, and hence the possibility (31) is excluded.

Let us now consider the LSND data [1]. In this case we have

$$|\Delta m^2_{21}L/4E|, |\Delta m^2_{43}L/4E| \ll 1 \tag{40}$$

so we obtain

$$\epsilon = 4\epsilon|U_{e3}U^*_{\mu 3} + U_{e4}U^*_{\mu 4}|^2 f_{\text{LSND}}\left(\Delta m^2_{31}\right), \tag{41}$$

with

$$f_{\text{LSND}}\left(\Delta m^2_{31}\right) \equiv \frac{1}{\epsilon}\left\langle \sin^2\left(\frac{\Delta m^2 L}{4E}\right)\right\rangle_{\mu \to e} \tag{42}$$
$$\begin{cases} = 1/\sin^2 2\theta_{\text{LSND}}(\Delta m^2) & \text{for } \Delta m^2 > 0.1\text{eV}^2 \\ \simeq \text{const.}(\Delta m^2)^2 & \text{for } \Delta m^2 < 0.1\text{eV}^2, \end{cases}$$

where $\sin^2 2\theta_{\text{LSND}}(\Delta m^2)$ is the value in the allowed region in the $(\Delta m^2, \sin^2 2\theta)$ plot in [1], and $25 \lesssim f_{\text{LSND}}(\Delta m^2_{31}) \lesssim 580$. Using the conditions on s_{13}, s_{14}, s_{23}, s_{24}, we obtain

$$1 \simeq 4\left|\frac{1}{\sqrt{2}}(s_{13} + s_{14})\right|^2 f_{\text{LSND}}\left(\Delta m^2_{31}\right) \tag{43}$$

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and hence

$$9° \gtrsim \max(\theta_{13}, \theta_{14}) \gtrsim 0.8°, \tag{44}$$

where we have also shown the upper bound (28).

3.2 Mass Pattern (ii)

We now show that the mass patterns (10) – (13) are excluded by reactor and accelerator experiments. The arguments against the four cases are exactly the same, so for simplicity we will consider the case (10), where $\Delta m_{4j}^2 \simeq \Delta m_{\text{LSND}}^2$ for $j = 1, 2, 3$. If the flight length L of neutrinos in the experiment satisfies $\Delta E_{41} L \sim \mathcal{O}(1)$, then the oscillation probabilities are given by

$$P(\nu_\alpha \rightarrow \nu_\beta) \simeq 4|U_{\alpha 4}|^2 |U_{\beta 4}|^2 \sin^2 \left(\frac{\Delta E_{41} L}{2} \right) \tag{45}$$

$$P(\nu_\alpha \rightarrow \nu_\alpha) \simeq 1 - 4|U_{\alpha 4}|^2 \left(1 - |U_{\alpha 4}|^2 \right) \sin^2 \left(\frac{\Delta E_{41} L}{2} \right), \tag{46}$$

where CP violating terms have been dropped in $P(\nu_\alpha \rightarrow \nu_\beta)$. Exactly with the same arguments as in the previous subsection, the data by Bugey in this case implies

$$|U_{e4}|^2 < \frac{1}{2} \left[1 - \sqrt{1 - 1/f_{\text{Bugey}} \left(\Delta m_{41}^2 \right)} \right], \tag{47}$$

the CDHSW data leads to

$$|U_{\mu 4}|^2 < \frac{1}{2} \left[1 - \sqrt{1 - 1/f_{\text{CDHSW}} \left(\Delta m_{41}^2 \right)} \right], \tag{48}$$

and the LSND data to

$$1 = 4|U_{e4}|^2 |U_{\mu 4}|^2 f_{\text{LSND}} \left(\Delta m_{41}^2 \right). \tag{49}$$

So Δm_{41}^2 has to satisfy the following constraint:

$$4|U_{e4}|^2|U_{\mu 4}|^2$$

$$= \frac{1}{f_{\text{LSND}}\left(\Delta m_{41}^2\right)}$$

$$< \left[1 - \sqrt{1 - \frac{1}{f_{\text{Bugey}}\left(\Delta m_{41}^2\right)}}\right]\left[1 - \sqrt{1 - \frac{1}{f_{\text{CDHSW}}\left(\Delta m_{41}^2\right)}}\right]. \quad (50)$$

It turns out that the condition (50) is not satisfied for any Δm_{41}^2 in the entire region $0.27\text{eV}^2 \lesssim \Delta m_{41}^2 \lesssim 2.3\text{eV}^2$. Hence the mass patterns (10) − (13) are excluded by reactor and accelerator experiments.

4 Big Bang Nucleosynthesis

Let us now discuss the constraints by cosmology. It has been shown [45][46][47] that the transitions $\nu_\alpha \to \nu_s$ ($\alpha = e, \mu, \tau$) have to be suppressed strongly for big bang nucleosynthesis to be consistent with the standard scenario, if the number N_ν of light neutrinos is less than four.[5] Here we follow the argument of [45][46] and give a rough estimate on the allowed region of the mixing angles without detailed numerical calculations. Except in section 7, we will assume $N_\nu < 4$ in the following sections. We will show in section 7 that if $N_\nu \geq 4$ we have much weaker constraints by big bang nucleosynthesis compared to the case for $N_\nu < 4$.

To get a rough idea on the magnitude of the effective mixing angle of neutrino oscillations, let us consider the temperature dependence of the difference ΔE of the kinetic term and the potential term V [45][46][47].

$$\Delta E = \frac{\Delta m^2}{6.3T} \quad (51)$$

$$V = -\frac{\sqrt{2}(7/90)^2\pi^6(2 + \cos^2\theta_W)G_F}{\zeta(3)m_W^2}T^5. \quad (52)$$

---

[5]For recent discussions on N_ν, see [48][49] and references therein.

From (51) and (52) we have

$$\frac{V}{\Delta E} \simeq -\left(\frac{T}{13\text{MeV}}\right)^6 \left(\frac{\Delta m^2}{\text{eV}^2}\right)^{-1}. \tag{53}$$

In case of the two flavor neutrino oscillation with a mixing angle θ, the interaction rate of sterile neutrinos through neutrino oscillations is given by

$$\left(\frac{\Gamma_{\nu_s}}{H}\right)_{2-\text{flavor}} = \sin^2 2\theta_M \ \sin^2\left(\mathcal{E}\ell_{\text{coll}}\right)\frac{\Gamma_\nu}{H}, \tag{54}$$

where θ_M is the effective mixing angle in the presence of interactions defined by

$$\sin^2 2\theta_M \equiv \frac{\sin^2 2\theta}{(\cos 2\theta - V/\Delta E)^2 + \sin^2 2\theta}, \tag{55}$$

$$H \equiv \left[\frac{4\pi^3}{45}\frac{g_*(T)}{m_{\text{pl}}^2}T^4\right]^{1/2} \tag{56}$$

is the Hubble parameter, $g_*(T) = 10.75$ is the effective degrees of freedom of particles for $m_e \lesssim T \lesssim m_\mu$,

$$\mathcal{E} \equiv \frac{1}{2}\left(\Delta E^2 + V^2 - 2V\Delta E\cos 2\theta\right)^{1/2} \tag{57}$$

is the eigenvalue of the mass matrix

$$e^{i\sigma_2\theta}\text{diag}\left(E_1, E_2\right)e^{-i\sigma_2\theta} + \text{diag}\left(V, 0\right), \tag{58}$$

$$\ell_{\text{coll}} \equiv 1/\Gamma_\nu = 1/C(T)G_F^2 T^5 \tag{59}$$

is the collision length of neutrino, Γ_ν is an interaction rate for active neutrinos, $C(T) \simeq 0.5$ for $m_e \lesssim T \lesssim m_\mu$, and

$$\frac{\Gamma_\nu}{H} \simeq \left(\frac{T}{1.9\text{MeV}}\right)^3. \tag{60}$$

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Notice that there is no enhancement of oscillations due to the MSW mechanism [10] in the present case, because $\theta_M < \theta$ follows from $V/\Delta E < 0$ in (55). Since we are interested in the mass squared differences $\Delta m_{21}^2 \sim \mathcal{O}(10^{-5}\text{eV}^2)$ or $\mathcal{O}(10^{-10}\text{eV}^2)$, $\Delta m_{43}^2 \sim \mathcal{O}(10^{-2}\text{eV}^2)$, $\Delta m_{31}^2 \sim \mathcal{O}(1\text{eV}^2)$, we have to consider situations where $\Delta E_{jk} = \Delta m_{jk}^2/6.3T$ becomes comparable to the absolute value $|V|$ of the potential, and this condition is satisfied for the critical temperature $T \sim 2\text{MeV}$ (or 0.3MeV), 8MeV, 15MeV, respectively. Off these regions of T, the problem becomes simpler. If $T \gg 25\text{MeV}$, $|V/\Delta E|$ becomes very large and the effective mixing angle is small, so that we have

$$
\begin{aligned}
\left(\frac{\Gamma_{\nu_s}}{H}\right)_{2-\text{flavor}} &\leq \sin^2 2\theta_M \, \frac{\Gamma_\nu}{H} \\
&\sim \sin^2 2\theta \, \left(\frac{\Delta E}{V}\right)^2 \left(\frac{T}{1.9\text{MeV}}\right)^3 \\
&\sim \sin^2 2\theta \, \left(\frac{T}{25\text{MeV}}\right)^{-9} \left(\frac{\Delta m^2}{\text{eV}^2}\right)^2 \ll 1 \\
&\qquad \text{for } \gg 25\text{MeV}.
\end{aligned}
\tag{61}
$$

On the other hand, if $T \ll 1\text{MeV}$, $|V/\Delta E|$ becomes very small and this case is reduced to the oscillations in vacuum:

$$
\left(\frac{\Gamma_{\nu_s}}{H}\right)_{2-\text{flavor}} \lesssim \sin^2 2\theta \, \left(\frac{T}{1.9\text{MeV}}\right)^3 \ll 1
$$
$$
\text{for } \ll 1\text{MeV}.
\tag{62}
$$

In the above extreme cases there is little transition $\nu_s \to \nu_\alpha$ ($\alpha = e, \mu, \tau$) and we have no problem with big bang nucleosynthesis. So in the following discussions we will discuss only the region of temperature $m_e \lesssim T \lesssim m_\mu$. Since $\Delta E \ell_{\text{coll}}/2 \sim 250|\Delta E/V|$ for any region of T, the term $\sin^2(\mathcal{E}\ell_{\text{coll}})$ in the formula of neutrino oscillations can be put to $1/2$ after averaging over rapid oscillations, as long as the temperature under consideration satisfies $|\Delta E/V| \sim \mathcal{O}(1)$.

To analyze oscillations among four species of neutrinos, it is necessary to diagonalize the mass matrix

$$\mathcal{M} \equiv U \mathrm{diag}\left(E_1, E_2, E_3, E_4\right) U^{-1} + \mathrm{diag}\left(V, cV, cV, 0\right), \tag{63}$$

where we have taken into account the fact that ν_e has both charged and neutral current interactions while ν_μ and ν_τ have only neutral ones for $m_e \lesssim t \lesssim m_\mu$, and

$$c \equiv \cos^2 \theta_W / (2 + \cos^2 \theta_W) \simeq 0.28 \tag{64}$$

is the ratio of neutral current interactions to the total contribution. Here we consider three cases where the potential term becomes comparable to the energy difference $|V| \simeq \Delta E_{jk} = \Delta m_{jk}^2 / 6.3T$ for $(j, k) = (2, 1), (4, 3), (3, 1)$. This corresponds to the critical temperature $T \simeq 2\mathrm{MeV}$ (or $0.3\mathrm{MeV}$), $8\mathrm{MeV}$, $15\mathrm{MeV}$, respectively. From the constraint (28) by reactor experiments we know that U_{e3} and U_{e4} are small, so we take $U_{e3} = U_{e4} = 0$ in what follows for simplicity. Since we assume $N_\nu < 4$, sterile neutrinos should never have been in thermal equilibrium, and we demand that the interaction rate Γ_{ν_s} of sterile neutrinos through neutrino oscillations be smaller than the Hubble parameter H:

$$
\begin{aligned}
\Gamma_{\nu_s} &= \sum_{\alpha=e,\mu,\tau} P(\nu_s \rightarrow \nu_\alpha) \Gamma_{\nu_\alpha} \\
&= \left[P(\nu_s \rightarrow \nu_e) + B \left\{ P(\nu_s \rightarrow \nu_\mu) + P(\nu_s \rightarrow \nu_\tau) \right\} \right] \Gamma_\nu \\
&< H, \tag{65}
\end{aligned}
$$

where

$$B \equiv \left(\frac{1 - 4\sin^2 \theta_W + 8\sin^4 \theta_W}{1 + 4\sin^2 \theta_W + 8\sin^4 \theta_W} \right) \simeq 0.21. \tag{66}$$

In (65) both charged and neutral current interactions are taken into account for ν_e, while only neutral ones are included for ν_μ and ν_τ.

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Let us first consider the situation where $|V| \simeq \Delta E_{21}$. For the vacuum oscillation solution, we can give the same argument and arrive at the same conclusion as the MSW ones. In what follows, therefore, we will discuss for simplicity the case of the MSW solutions, which imply a critical temperature $T \simeq 2\text{MeV}$. Now if $T \simeq 2\text{MeV}$, the mass matrix (63) is diagonalized as

$$\mathcal{M} \simeq U U_M \left[(E_1 + cV)\mathbf{1}_4 + \text{diag}\,(\lambda_1, \lambda_2, \Delta E_{31}, \Delta E_{41}) \right] U_M^{-1} U^{-1}, \qquad (67)$$

where

$$\lambda_1, \lambda_2 = \Delta E_{21} \mp \frac{1}{2} \left[\Delta E_{21}^2 - 2\Delta E_{21}(1 - cA)V \cos 2\theta_\odot + (1 - cA)V^2 \right]^{1/2}, \qquad (68)$$

$$U_M \equiv \begin{pmatrix} e^{i\sigma_2 \theta_{12}^M} & \mathbf{0} \\ \mathbf{0} & \mathbf{1}_2 \end{pmatrix}, \qquad (69)$$

$$A \equiv |U_{s3}|^2 + |U_{s4}|^2 \qquad (70)$$

with

$$\tan 2\theta_{12}^M \equiv \frac{(1 - cA)V \sin 2\theta_\odot}{\Delta E_{21} - (1 - cA)V \cos 2\theta_\odot}. \qquad (71)$$

In this case difference of any two eigenvalues of $\mathcal{M}$ is large compared to the collision length ℓ_{coll}, so the factor $\sin^2\left(\mathcal{E}\ell_{\text{coll}}\right)$ can be put to $1/2$, and the probabilities are given by

$$P(\nu_s \to \nu_e) \simeq \frac{1 - A}{2} \sin^2 2(\theta_{12}^M + \theta_\odot) \qquad (72)$$

$$P(\nu_s \to \nu_\mu) + P(\nu_s \to \nu_\tau)$$

$$\simeq A(1 - A)\left\{ 2 - \frac{1}{2}\sin^2 2(\theta_{12}^M + \theta_\odot) \right\} + \frac{A^2}{2}\sin^2 2\phi, \quad (73)$$

where

$$\sin^2 2(\theta_{12}^M + \theta_\odot) \equiv \frac{\sin^2 2\theta_\odot}{1 - 2(1 - cA)V \cos 2\theta_\odot / \Delta E_{21} + (1 - cA)^2 V^2 / \Delta E_{21}^2}, \qquad (74)$$

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and we have parametrized the matrix elements as

$$\begin{pmatrix} U_{s3} \\ U_{s4} \end{pmatrix} \equiv A^{1/2} e^{i(\chi + \sigma_3 \psi)/2 - i\sigma_2 \phi} \begin{pmatrix} 1 \\ 0 \end{pmatrix} \tag{75}$$

with

$$\chi \equiv \arg(U_{s3} U_{s4}), \tag{76}$$

$$\psi \equiv \arg(U_{s3}/U_{s4}), \tag{77}$$

$$\phi \equiv \tan^{-1} |U_{s4}/U_{s3}|. \tag{78}$$

Thus we have the interaction rate of sterile neutrinos

$$\begin{aligned} \Gamma_{\nu_s} &= [P(\nu_s \to \nu_e) + B\{P(\nu_s \to \nu_\mu) + P(\nu_s \to \nu_\tau)\}] \Gamma_\nu \\ &\simeq \left[B\{2A(1-A) + \frac{A^2}{2} \sin^2 2\phi\} + \{\frac{1-A}{2} - \frac{B}{2} A(1-A)\} \sin^2 2(\theta_{12}^M + \theta_\odot) \right] \Gamma_\nu \\ &\quad \text{for } T \sim 2\text{MeV}. \end{aligned} \tag{79}$$

If $T \simeq 8\text{MeV}$ (i.e., $|V| \simeq \Delta E_{43}$), the mass matrix (63) is diagonalized as

$$\mathcal{M} \simeq U U_M \left[(E_3 + cV)\mathbf{1}_4 + \text{diag}\,(-\Delta E_{31}, -\Delta E_{32}, \lambda_3, \lambda_4) \right] U_M^{-1} U^{-1}, \tag{80}$$

where

$$\lambda_3, \lambda_4 = \Delta E_{43} \mp \frac{1}{2} \left[\Delta E_{43}^2 + 2\Delta E_{43} cAV \cos 2\theta_\odot + c^2 A^2 V^2 \right]^{1/2}, \tag{81}$$

$$U_M \equiv \begin{pmatrix} \mathbf{1}_2 & \mathbf{0} \\ \mathbf{0} & e^{i\sigma_2 \theta_{34}^M} \end{pmatrix}, \tag{82}$$

with

$$\tan 2\theta_{34}^M \equiv \frac{cAV \sin 2\phi}{\Delta E_{34} + cAV \cos 2\phi}. \tag{83}$$

All differences of two eigenvalues of $\mathcal{M}$ except ΔE_{21} are large compared to the collision length ℓ_{coll}, and we put all the factors $\sin^2 (\mathcal{E} \ell_{\text{coll}})$ to 1/2 but set

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$\sin^2\left(\Delta E_{21}\ell_{\text{coll}}\right)$ to zero. The probabilities are given by

$$P(\nu_s \to \nu_e) \simeq 0 \tag{84}$$

$$P(\nu_s \to \nu_\mu) + P(\nu_s \to \nu_\tau)$$
$$\simeq 2A(1-A) + \frac{A^2}{2}\sin^2 2(\phi - \theta_{34}^M), \tag{85}$$

where

$$\sin^2 2(\phi - \theta_{34}^M) \equiv \frac{\sin^2 2\phi}{1 - 2cAV\cos 2\phi/\Delta E_{43} + c^2 A^2 V^2/\Delta E_{43}^2}. \tag{86}$$

The interaction rate of sterile neutrinos is given by

$$\Gamma_{\nu_s} \simeq B\left[2A(1-A) + \frac{A^2}{2}\sin^2 2(\phi - \theta_{34}^M)\right]\Gamma_\nu.$$
$$\text{for } T \sim 8\text{MeV} \tag{87}$$

If $T \simeq 15\text{MeV}$ (i.e., $|V| \simeq \Delta E_{31}$), the mass matrix (63) is diagonalized as

$$\mathcal{M} \simeq UU_M\left[E_1\mathbf{1}_4 + \text{diag}\left(V, -\Delta E_{31}, \lambda_5, \lambda_6\right)\right]U_M^{-1}U^{-1}, \tag{88}$$

where

$$\lambda_5, \lambda_6 = \frac{-\Delta E_{31} + cV}{2} \pm \frac{1}{2}\left[(\Delta E_{31}^2 - cV)^2 + 4cAV\Delta E_{31}\right]^{1/2}, \tag{89}$$

$$U_M \equiv \begin{pmatrix} U_{e1} & 0 & \alpha_+\begin{pmatrix} U_{s1}^* \\ U_{s2}^* \end{pmatrix} & \alpha_-\begin{pmatrix} U_{s1}^* \\ U_{s2}^* \end{pmatrix} \\ U_{e2} & 0 & & \\ 0 & A^{-\frac{1}{2}}\begin{pmatrix} U_{s4} \\ -U_{s3} \end{pmatrix} & \beta_+\begin{pmatrix} U_{s3}^* \\ U_{s4}^* \end{pmatrix} & \alpha_-\begin{pmatrix} U_{s3}^* \\ U_{s4}^* \end{pmatrix} \end{pmatrix}. \tag{90}$$

In (90) $\alpha_\pm, \beta_\pm$ satisfy the normalization conditions

$$A|\beta|^2 + (1-A)|\alpha|^2 = 1, \tag{91}$$

and $x_\pm \equiv \beta_\pm/\alpha_\pm$ are the two roots of a quadratic equation

$$Ax^2 + (1 - 2A + \Delta E_{31}/cV)x - (1-A) = 0. \tag{92}$$

From (90) it turns out that $(UU_M)_{e1} = 1$, $(UU_M)_{ej} = 0$ for $j = 2, 3, 4$,
$(UU_M)_{sj} = 0$ for $j = 3, 4$, and the only combination which appears in the formula of probability with non-vanishing coefficients $(UU_M)_{sj}(UU_M)^*_{sk}(UU_M)^*_{ej}$
$(UU_M)_{ek}$ is $(j, k) = (3, 4)$, and the difference of these two eigenvalues of $\mathcal{M}$
is large compared to the collision length ℓ_{coll}. Hence we again put all the
factors $\sin^2(\mathcal{E}\ell_{\text{coll}})$ to $1/2$, and we obtain the probabilities

$$P(\nu_s \to \nu_e) \simeq 0 \tag{93}$$

$$P(\nu_s \to \nu_\mu) + P(\nu_s \to \nu_\tau)$$

$$\simeq 2|\alpha_+|^2|\alpha_-|^2|1 - A + x_+ A|^2|1 - A + x_- A|^2 \tag{94}$$

$$= \frac{2A(1 - A)}{1 + 2(1 - 2A)cV/\Delta E_{31} + c^2V^2/\Delta E_{31}^2}. \tag{95}$$

The interaction rate of sterile neutrinos is then given by

$$\Gamma_{\nu_s} \simeq \frac{2c^2A(1 - A)}{1 + 2(1 - 2A)cV/\Delta E_{31} + c^2V^2/\Delta E_{31}^2}\Gamma_\nu.$$
$$\text{for } T \sim 15\text{MeV} \tag{96}$$

Combining the results (79), (87) and (96) for three cases we have the following
formula

$$\frac{\Gamma_{\nu_s}}{H} \simeq \left(\frac{T}{1.8\text{MeV}}\right)^3 \left[\frac{2BA(1 - A)}{1 + 2(1 - 2A)cV/\Delta E_{31} + c^2V^2/\Delta E_{31}^2}\right.$$

$$+ \frac{BA^2\sin^2 2\phi/2}{1 - 2cAV\cos 2\phi/\Delta E_{43} + c^2A^2V^2/\Delta E_{43}^2}$$

$$+ \left.\frac{(1 - A)(1 - BA)\sin^2 2\theta_\odot/2}{1 - 2(1 - cA)V\cos 2\theta_\odot/\Delta E_{21} + (1 - cA)^2V^2/\Delta E_{21}^2}\right], \tag{97}$$

which approximately holds for any temperature $m_e \lesssim T \lesssim m_\mu$ [6]. As in [46]
we look for the extremum of the expression Γ_{ν_s}/H with respect to T. The

[6]Notice that results (79), (87), (96) are reproduced for each case in (97): the first term
$\to 2c^2A(1 - A)$, the second $\to c^2A^2\sin^2 2\phi/2$ for $T \sim 2\text{MeV}$ where $V/\Delta E_{31}, V/\Delta E_{43} \to 0$,
the first term $\to 2c^2A(1 - A)$, the third $\sim (T/2\text{MeV})^{-12} \to 0$ for $T \sim 8\text{MeV}$
where $V/\Delta E_{31} \to 0, |V/\Delta E_{21}| \to$ large, the second $\sim (T/8\text{MeV})^{-12} \to 0$, the third
$\sim (T/2\text{MeV})^{-12} \to 0$ for $T \sim 15\text{MeV}$ where $|V/\Delta E_{31}|, |V/\Delta E_{43}| \to$ large.

extrema of the three terms in (97) are attained at different values of T which are far apart, so we can discuss the extremum of the total quantity (97) by examining each term separately. Each term in (97) has the following maximum value

$$
\max_T \text{ (1st term)} \simeq \left(\frac{15}{1.8}\right)^3 \left(\frac{\Delta m_{31}^2}{\text{eV}^2}\right)^{1/2} \frac{3B}{2c^{1/2}}
$$

$$
\times \frac{A(1-A)\left[[1/3 + ((1-2A)/3)^2]^{1/2} - (1-2A)/3\right]^{1/2}}{1 - (1-2A)^2/3 + (1-2A)\left[1/3 + ((1-2A)/3)^2\right]^{1/2}} \tag{98}
$$

$$
\max_T \text{ (2nd term)} \simeq \left(\frac{7.7}{1.8}\right)^3 \left(\frac{\Delta m_{43}^2}{2 \times 10^{-2}\text{eV}^2}\right)^{1/2} \frac{3A^{3/2}B}{8c^{1/2}}
$$

$$
\times \frac{\sin^2 2\phi \left[[1/3 + (\cos 2\phi/3)^2]^{1/2} + \cos 2\phi/3\right]^{1/2}}{1 + \cos^2 2\phi/3 + \cos 2\phi \left[1/3 + (\cos 2\phi/3)^2\right]^{1/2}} \tag{99}
$$

$$
\max_T \text{ (3rd term)} \simeq \left(\frac{1.9}{1.8}\right)^3 \left(\frac{\Delta m_{21}^2}{10^{-5}\text{eV}^2}\right)^{1/2} \frac{3(1-A)(1-BA)}{8(1-cA)^{1/2}}
$$

$$
\times \frac{\sin^2 2\theta_\odot \left[[1/3 + (\cos 2\theta_\odot/3)^2]^{1/2} - \cos 2\theta_\odot/3\right]^{1/2}}{1 - \cos^2 2\theta_\odot/3 + \cos 2\theta_\odot \left[1/3 + (\cos 2\theta_\odot/3)^2\right]^{1/2}}. \tag{100}
$$

The term in the second line in (98) is a monotonically increasing function in A, so the first term in (97) gives a necessary condition

$$
A \lesssim \frac{8}{3\sqrt{3}} \left(\frac{1.8}{15}\right)^3 \frac{c^{1/2}}{B} \left(\frac{\Delta m_{31}^2}{\text{eV}^2}\right)^{-1/2} \lesssim 1.3 \times 10^{-2}, \tag{101}
$$

where we have used the property that the term in the second line in (98) is approximated as $\sqrt{3}A/4$ as $A \to 0$ and the lower bound $\Delta m_{31}^2 \gtrsim 0.27\text{eV}^2$ in (1). It is straightforward to see that the term in the second line in (99) is less $4/3$ for any value of $\cos 2\phi$, and it follows from (101)

$$
\max_T \text{ (2nd term)} \lesssim 2 \times 10^{-2} \left(\frac{\Delta m_{43}^2}{2 \times 10^{-2}\text{eV}^2}\right)^{1/2} \ll 1, \tag{102}
$$

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where we have used the combined Kamiokande results of sub-GeV [15] and multi-GeV [16] atmospheric neutrinos, which suggest $5 \times 10^{-3} \mathrm{eV}^2 \lesssim \Delta m_{43}^2 \lesssim 3 \times 10^{-2} \mathrm{eV}^2$. So the contribution coming from $T \sim 8 \mathrm{MeV}$ never brings ν_s into thermal equilibrium, as long as $\max_T(\text{1st term}) < 1$ in (98) is satisfied. The term in the second line in (100) is approximately less than 0.54 for $\sin^2 2\theta_\odot \leq 0.9$ which is satisfied by both the MSW solutions. Thus we have

$$\max_T (\text{3rd term}) \quad \lesssim \quad 0.24 \left(\frac{\Delta m_{21}^2}{10^{-5} \mathrm{eV}^2} \right)^{1/2}, \tag{103}$$

so ν_s is not brought into thermal equilibrium in case of the MSW solutions, since $\Delta m_{21}^2 \lesssim 9 \times 10^{-5} \mathrm{eV}^2$ is satisfied by both the MSW solutions. Exactly by the same argument as above, we can show that the vacuum oscillation solution is allowed, as far as (100) is concerned[7]. From (28) and (101) it follows

$$|U_{s3}|^2, |U_{s4}|^2 \lesssim 1.3 \times 10^{-2} \tag{104}$$

From (37) we have $0.33 \lesssim c_{24}^2 \lesssim 0.82$, so that

$$|U_{s4}|^2 \simeq c_{24}^2 s_{34}^2 \gtrsim 0.33 \ s_{34}^2, \tag{105}$$

where we have used the relations

$$U_{s3} = c_{13}c_{34}s_{23}e^{-i\delta_1} - c_{24}s_{13}s_{14}s_{34}e^{i(\delta_2-\delta_3)} - c_{13}c_{23}s_{24}s_{34} \tag{106}$$
$$U_{s4} = c_{14}c_{24}s_{34}. \tag{107}$$

From (104) and (105) we conclude

$$s_{34}^2 \lesssim 1.3 \times 10^{-2}/0.33 \simeq 3.8 \times 10^{-2}, \tag{108}$$

---

[7]The critical temperature for the vacuum oscillation solution is $T \simeq 0.3 \mathrm{MeV}$, so the effective degrees $g_*(T)$ of freedom becomes smaller than 10.75. Nevertheless, it turns out that $\max_T (\text{3rd term}) \lesssim \mathcal{O}(10^{-1}) \times (\Delta m_{21}^2/10^{-5} \mathrm{eV}^2)^{1/2}$ is satisfied also for the vacuum solution and this maximum value is obviously much less than 1.

or

$$\theta_{34} \lesssim 11^\circ. \tag{109}$$

From (28) and (109), we have

$$|U_{s3}|^2 \simeq s_{23}^2 \lesssim 1.3 \times 10^{-2} \tag{110}$$

or

$$\theta_{23} \lesssim 6^\circ. \tag{111}$$

To summarize, for $N_\nu < 4$ the mixing matrix looks like

$$V_{KM} \sim \begin{pmatrix} c_\odot & s_\odot & \epsilon & \epsilon \\ \epsilon & \epsilon & c_{\text{atm}} & s_{\text{atm}} \\ \epsilon & \epsilon & -s_{\text{atm}} & c_{\text{atm}} \\ -s_\odot & c_\odot & \epsilon & \epsilon \end{pmatrix}, \tag{112}$$

where we have defined $\theta_\odot \equiv \theta_{12}$, $\theta_{\text{atm}} \equiv \theta_{24}$, ϵ stands for a small number, and all the CP violating phases have been dropped out because $|\theta_{13}|, |\theta_{14}|, |\theta_{23}|, |\theta_{34}| \ll 1$. The basic reason that we have a strong constraint such as (112) is because we have demanded that one of the mass scales is given by Δm_{LSND}^2, which implies a severe bound on Γ_{ν_s}/H from big bang nucleosynthesis.

It was pointed out in [45] that $\nu_e \leftrightarrow \nu_s$ oscillation is potentially dangerous because it could change the density of ν_e at $T \lesssim T_\nu$ and therefore could increase the nucleon freeze-out temperature. To avoid this effect, it is necessary for the probability $P(\nu_e \to \nu_s)$ to satisfy

$$\frac{1}{2}\left(1 + (1 - P(\nu_e \to \nu_s))\right) = \left(\frac{10.75}{10.75 + \frac{7}{4}(N_\nu - 3)}\right)^{1/2} > 0.93 \tag{113}$$

where we have substituted $G_F^2 \to G_F^2(1 + (1 - P(\nu_e \to \nu_s)))/2$ in the equilibrium condition

$$G_F^2 T^5 \simeq g_*(T)^{1/2} T^2/m_{\text{pl}} \tag{114}$$

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in case of $\nu_e \leftrightarrow \nu_s$ oscillation, while we have used $N_\nu < 3.9$ [48] and have put $g_*(T) = 10.75 \rightarrow 10.75 + (7/4)(N_\nu - 3) < 10.75 + (7/4) \times 0.9$ on the right hand side of (114). As we have seen above, the only case where $\nu_e \leftrightarrow \nu_s$ oscillation occurs is $T \sim 2\text{MeV}$ (See (72)), and using (113) and the condition $A \ll 1$ we have

$$\sin^2 2(\theta_{12}^M + \theta_\odot) < 0.26. \tag{115}$$

The numerical value here differs slightly from the one in [45], but in either case we arrive at the conclusion that the large-angle MSW solution is excluded, since (74) exceeds 0.26 for some region of T for $0.6 \lesssim \sin^2 2\theta_\odot \lesssim 0.9$. Note that this argument does not apply to the vacuum solution, because the collision length is much shorter than the oscillation length. We note in passing that the large-angle MSW solution for $\nu_e \leftrightarrow \nu_s$ is excluded without referring to big bang nucleosynthesis, by combining all the data of solar neutrino experiments, the earth effect and Kamiokande day-night effect [12][13].

It has been shown in the two flavor analysis [14] that the vacuum oscillation solution for the channel $\nu_e \leftrightarrow \nu_s$ is excluded at 95 % confidence level if the standard solar model is taken for granted. The mixing matrix (112) tells us that our model in case of $N_\nu < 4$ is described approximately by two-flavor mixings $\nu_e \leftrightarrow \nu_s$ and $\nu_\mu \leftrightarrow \nu_\tau$, so we conclude that the vacuum oscillation solution is also excluded in our scenario. Hence we are left only with the small-angle MSW solution for $N_\nu < 4$. This is consistent with a naive anticipation that the small-angle MSW solution is the most preferable scenario, because it fits the solar neutrino data best [11]–[14].

5   Double β Decay

Assuming that neutrinos are of Majorana type, let us now consider the implication to the neutrinoless double β decay experiments. To discuss neu-

trinoless double β decays, we have to consider the magnitudes of neutrino masses instead of mass squared differences. From our assumption on the mass hierarchy $\Delta m_{21}^2, \Delta m_{43}^2 \ll \Delta m_{31}^2, \Delta m_{41}^2, \Delta m_{32}^2, \Delta m_{42}^2$, we have

$$m_1 \simeq m_2, \ m_3 \simeq m_4 \simeq \sqrt{m_1^2 + \Delta m_{31}^2}. \tag{116}$$

From the form of the mixing matrix (112) the two pairs (ν_e, ν_s) and (ν_μ, ν_τ) have basically no mixing with each other, and we have

$$(m_{\nu_e}, m_{\nu_s}) \simeq m_1, \qquad (m_{\nu_\mu}, m_{\nu_\tau}) \simeq \sqrt{m_1^2 + \Delta m_{31}^2}$$
$$\text{for mass pattern (ia).} \tag{117}$$

In case of the mass pattern (9), the mixing matrix looks like

$$V_{KM} \sim \begin{pmatrix} \epsilon & \epsilon & c_\odot & s_\odot \\ c_{\text{atm}} & s_{\text{atm}} & \epsilon & \epsilon \\ -s_{\text{atm}} & c_{\text{atm}} & \epsilon & \epsilon \\ \epsilon & \epsilon & -s_\odot & c_\odot \end{pmatrix}, \tag{118}$$

so that we have

$$(m_{\nu_e}, m_{\nu_s}) \simeq \sqrt{m_1^2 + \Delta m_{31}^2}, \qquad (m_{\nu_\mu}, m_{\nu_\tau}) \simeq m_1$$
$$\text{for mass pattern (ib).} \tag{119}$$

It is well known [37]–[41] that the CP violating phases α, β, γ in (3) cannot be absorbed by redefinition of the neutrino fields. So the effective electron neutrino mass, which is the e-e component of the mass matrix, is given by

$$\begin{aligned} \langle m_{\nu_e} \rangle &\equiv \left| \left[V_{KM} \, D \, \text{diag} \, (m_j) \, D \, V_{KM}^T \right]_{ee} \right| \\ &\simeq \begin{cases} m_1 \left| e^{i\alpha} c_\odot^2 + e^{-i\alpha} s_\odot^2 \right| \simeq m_1 \\ \qquad\qquad \text{for mass pattern (ia)} \\[2ex] \sqrt{m_1^2 + \Delta m_{31}^2} \left| e^{i\alpha} c_\odot^2 + e^{-i\alpha} s_\odot^2 \right| \simeq \sqrt{m_1^2 + \Delta m_{31}^2} \\ \qquad\qquad \text{for mass pattern (ib),} \end{cases} \end{aligned} \tag{120}$$

where $D \equiv e^{-i\sqrt{6}\gamma\lambda_{15}}e^{-i\beta\lambda_8}e^{-i\alpha\gamma\lambda_3}$ is a diagonal matrix and we have used (117), (119) and the condition $\sin^2 2\theta_\odot \simeq 6 \times 10^{-3}$ for the small-angle MSW solution. The present upper bound on $\langle m_{\nu_e} \rangle$ from neutrinoless double β decay experiments is [50][51]

$$\langle m_{\nu_e} \rangle < 0.68\text{eV}. \tag{121}$$

From (120), (121), (117) and (119) we obtain

$$m_1 \; \lesssim \; 0.68\text{eV} \qquad \text{for mass pattern (ia)}, \tag{122}$$

$$
\begin{aligned}
m_1 \; &\lesssim \; \sqrt{(0.68\text{eV})^2 - \Delta m_{31}^2} \\
&\lesssim \; 0.44\text{eV} \qquad \text{for mass pattern (ib)},
\end{aligned}
\tag{123}
$$

where in case of (123) we have used the lower bound in (1). Note that $\Delta m_{\text{LSND}}^2 \simeq \Delta m_{31}^2 < 0.46\text{eV}^2$ has to be satisfied in (123) for m_1 to be real.

6 Hot Dark Matter

It has been argued that a model with cold+hot dark matter with $5\text{eV} \lesssim \sum_\alpha m_{\nu_\alpha} \lesssim 7\text{eV}$ seems to be in agreement with observations, such as the anisotropy of the cosmic background radiations, correlation of galactic clusters, etc. [31][52], and efforts have been made to introduce sterile neutrinos to account for hot dark matter as well as other anomalies [29]–[34]. Here we examine the possibility that neutrinos could be hot dark matter while satisfying all the constraints that we have obtained in the previous sections.

Since we assume that sterile neutrinos have never been in thermal equilibrium, only three components of the mass matrix (117) or (119) contribute to the mass density. Using (122) and (123), therefore, we have the following

total mass bound:

$$
m_{\nu_e} + m_{\nu_\mu} + m_{\nu_\tau} \simeq
\begin{cases}
m_1 + 2\sqrt{m_1^2 + \Delta m_{31}^2} \lesssim 4.0\text{eV} \\
\quad \text{for mass pattern (ia)} \\
\\
2m_1 + \sqrt{m_1^2 + \Delta m_{31}^2} \lesssim 1.6\text{eV} \\
\quad \text{for mass pattern (ib).}
\end{cases}
\tag{124}
$$

From (124) we conclude that in either case neutrinos are not heavy enough to account for all the hot dark matter components.

7 $N_\nu \geq 4$

The possibility of $N_\nu \geq 4$ has been proposed recently [53], so we will also discuss this case for the sake of completeness. If $N_\nu \geq 4$, then sterile neutrinos should have been in thermal equilibrium, and we cannot put a strong constraint like the one for $N_\nu < 4$. The only condition which has to be satisfied for sterile neutrinos to be in thermal equilibrium is $A > 1.3 \times 10^{-2}$. We still have constraints from the reactor experiments which imply that θ_{13}, θ_{14} are small, but we can no longer say anything about the magnitude of θ_{23}, θ_{34}. The large-angle MSW solution is allowed in this case, as long as ν_s is in thermal equilibrium. As we noted earlier, however, the large-angle MSW and vacuum oscillation solutions are excluded for $\nu_e \leftrightarrow \nu_s$ in the two flavor analysis [12][13][14], so if U is very close to a direct sum of two mixing matrices with $\nu_e \leftrightarrow \nu_s$ and $\nu_\mu \leftrightarrow \nu_\tau$ channels then it contradicts with solar neutrino observations. The analysis of solar neutrino problem with four species of neutrinos would be extremely complicated[8], and it is yet to be seen

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[8]The explicit calculation for solar neutrino problem with three flavors has been performed recently in [26], assuming mass hierarchy.

under what conditions of mixing angles the large-angle MSW and vacuum oscillation solutions are allowed.

Here, for simplicity, let us consider an extreme case in which the oscillations take place mainly in the channels $\nu_e \leftrightarrow \nu_\tau$ and $\nu_\mu \leftrightarrow \nu_s$. This solution is consistent with all the solar neutrino observations [12][13][14], and ν_s is in thermal equilibrium because the large mixing angle of $\nu_\mu \leftrightarrow \nu_s$ suggested by the atmospheric neutrino anomaly [16] gives Γ_{ν_s}/H which is larger than one [54]. The masses of neutrinos in this case are given by

$$(m_{\nu_e}, m_{\nu_\tau}) \simeq m_1, \ (m_{\nu_\mu}, m_{\nu_s}) \simeq \sqrt{m_1^2 + \Delta m_{31}^2}$$
$$\text{for mass pattern (ia),} \tag{125}$$

$$(m_{\nu_e}, m_{\nu_\tau}) \simeq \sqrt{m_1^2 + \Delta m_{31}^2}, \ (m_{\nu_\mu}, m_{\nu_s}) \simeq m_1$$
$$\text{for mass pattern (ib).} \tag{126}$$

If the solar neutrino problem is solved by the small-angle MSW solution, then from (122) and (123) we have

$$
\begin{aligned}
m_{\nu_e} + m_{\nu_\mu} + m_{\nu_\tau} + m_{\nu_s} \ &\simeq\ 2m_1 + 2\sqrt{m_1^2 + \Delta m_{31}^2} \\
&\lesssim\ \begin{cases} 4.7\text{eV} & \text{for mass pattern (ia)} \\ 2.2\text{eV} & \text{for mass pattern (ib).} \end{cases}
\end{aligned}
\tag{127}
$$

So in this case neutrinos cannot account for all the hot dark matter components.

If we take the large-angle MSW solution, on the other hand, instead of (120) we have

$$
\langle m_{\nu_e} \rangle \simeq \begin{cases} m_1\sqrt{1 - \sin^2\alpha \sin^2 2\theta_\odot} \gtrsim 0.32 m_1 \\ \qquad\qquad\qquad\qquad \text{for mass pattern (ia)} \\ \\ \sqrt{m_1^2 + \Delta m_{31}^2}\sqrt{1 - \sin^2\alpha \sin^2 2\theta_\odot} \gtrsim 0.32\sqrt{m_1^2 + \Delta m_{31}^2} \\ \qquad\qquad\qquad\qquad \text{for mass pattern (ib),} \end{cases}
$$

$$\tag{128}$$

where we have used the condition $0.6 \lesssim \sin^2 2\theta_\odot \lesssim 0.9$ for the large-angle MSW solution. From (121) and (128) we have

$$
\begin{cases}
m_1 & \lesssim \quad 2.2\text{eV} \\
2m_1 + 2\sqrt{m_1^2 + \Delta m_{31}^2} & \lesssim \quad 9.6\text{eV}
\end{cases}
\qquad \text{for mass pattern (ia),}
$$
$$\tag{129}$$

$$
\begin{cases}
m_1 & \lesssim \quad 2.1\text{eV} \\
2m_1 + 2\sqrt{m_1^2 + \Delta m_{31}^2} & \lesssim \quad 8.5\text{eV}
\end{cases}
\qquad \text{for mass pattern (ib),}
$$
$$\tag{130}$$

so that neutrinos could explain all the hot dark matter. Conversely, if all the hot dark matter components are neutrinos in this case, then it follows

$$2m_1 + 2\sqrt{m_1^2 + \Delta m_{31}^2} \quad \gtrsim \quad 5\text{eV}, \tag{131}$$

$$m_1 \quad \gtrsim \quad 0.8\text{eV}, \tag{132}$$

$$\langle m_{\nu_e} \rangle \quad \gtrsim \quad
\begin{cases}
0.2\text{eV} & \text{for mass pattern (ia)} \\
0.4\text{eV} & \text{for mass pattern (ib),}
\end{cases}
\tag{133}
$$

so we will be able to observe neutrinoless double β decays in the future experiments.

If the solar neutrino problem is solved by the vacuum oscillation solution, we have

$$
\langle m_{\nu_e} \rangle \simeq
\begin{cases}
m_1\sqrt{1 - \sin^2 \alpha \sin^2 2\theta_\odot} \geq 0, \\
\qquad\qquad\qquad \text{for mass pattern (ia)} \\
\\
\sqrt{m_1^2 + \Delta m_{31}^2}\sqrt{1 - \sin^2 \alpha \sin^2 2\theta_\odot} \geq 0 \\
\qquad\qquad\qquad \text{for mass pattern (ib),}
\end{cases}
\tag{134}
$$

where we have used the condition $0.6 \lesssim \sin^2 2\theta_\odot \leq 1$ for the vacuum oscillation solution. If α and $2\theta_\odot$ are both very close to $\pi/2$, then m_1 can be arbitrarily large without contradicting with the bound (121) from the neutrinoless double β decay experiments. In this case, therefore, neutrinos could account for all the hot dark matter components, but there is no guarantee that we will be able to observe neutrinoless double β decays in future experiments.

8   Conclusions

In this paper we have performed a detailed analysis of the constraints on a model with three active and one sterile neutrinos, using the data of reactor and accelerator experiments, the solar and the atmospheric neutrino observations, and big bang nucleosynthesis. The mass pattern where three masses are degenerated is found to be inconsistent with reactor and accelerator experiments. If $N_\nu < 4$, then all the mixing angles are severely constrained and the mixing matrix is effectively split into 2×2 matrices with channels $\nu_e \leftrightarrow \nu_s$ and $\nu_\mu \leftrightarrow \nu_\tau$. In this case the large-angle MSW and vacuum oscillation solutions are excluded. Because of the constraints from neutrinoless double β decay experiments, neutrinos cannot explain all the hot dark matter components. For $N_\nu \geq 4$, we get fewer conditions on the mixing angles, and all the solutions to the solar neutrino problem are allowed. If we take either the large-angle MSW solution or the vacuum oscillation one, then these neutrinos can account for all the hot dark matter components. In case of the large-angle MSW solution for $N_\nu \geq 4$, we will be able to observe neutrinoless double β decays in near future. It is hoped that combined results of super-Kamiokande and SNO experiments can tell us about the existence of sterile neutrinos, the type of the solution to the solar neutrino problem, and

30

the mixing of sterile neutrinos [55].

References

[1] LSND Collaboration, C. Athanassopoulos et al., Phys. Rev. Lett. **75** (1995) 2650; preprint LA-UR-96-1326 (nucl-ex/9605001).

[2] J.E. Hill, Phys. Rev. Lett. **75** (1995) 2654.

[3] L. Borodovsky et al., Phys. Rev. Lett. **68** (1992) 274.

[4] B. Ackar et al., Nucl. Phys. **B434** (1995) 503.

[5] R. Davis Jr., *Proc. of the 6-th International Workshop on Neutrino Telescopes*, Venezia, March 1994, edited by M. Baldo Ceolin.

[6] Kamiokande Collaboration, K. S. Hirata et al., Phys. Rev. Lett. **63** (1989) 6; *ibid.* **65** (1990) 1297; *ibid.* **65** (1990) 1301; *ibid.* **66** (1991) 9; Phys. Rev. D **44** (1991) 2241.

[7] GALLEX Collaboration, P. Angelmann et al., Phys. Lett. **B285** (1992) 376; *ibid.* **B314** (1993) 445; *ibid.* **B327** (1994) 377; *ibid.* **B342** (1995) 440; *ibid.* **B357** (1995) 237.

[8] SAGE Collaboration, A.I. Abazov et al., Phys. Rev. Lett. **67** (1991) 3332; J.N. Abdurashitov et al., Phys. Lett. **B328** (1994) 234.

[9] See, e.g., J.N. Bahcall and R.K. Ulrich, Rev. Mod. Phys. **60** (1988) 297; J.N. Bahcall and M.H. Pinsonneault, Rev. Mod. Phys. **64** (1992) 885; J.N. Bahcall, R. Davis, Jr., P. Parker, A. Smirnov, R. Ulrich eds., *SOLAR NEUTRINOS: the first thirty years* Reading, Mass., Addison-Wesley, 1994 and references therein.

[10] S. P. Mikheyev and A. Smirnov, Nuovo Cim. **9C** (1986) 17; L. Wolfen-stein, Phys. Rev. **D17** (1978) 2369.

[11] X. Shi, D.N. Schramm and J.N. Bahcall, Phys. Rev. Lett. **69** (1992) 717; V. Barger, R.J.N. Phillips, and K. Whisnant, Phys. Rev. Lett. **69** (1992) 3135; S.A. Bludman, N. Hata, D.C. Kennedy and P.G. Langacker, Phys. Rev. **D47** (1993) 2220; L.M. Krauss, E. Gates and M. White, Phys. Lett. **299B** (1993) 95; P.I. Krastev and S.T. Petcov, Phys. Lett. **299B** (1993) 99; G.L. Fogli and E. Lisi, Astropart. Phys. **2** (1994) 91; P.I. Krastev and A.Yu. Smirnov, Phys. Lett. **B338** (1994) 282. G. Fiorentini, M. Lissia, G. Mezzorani, M. Moretti and D. Vignaud, Phys. Rev. D **49** 6298 (1994).

[12] N. Hata and P. Langacker, Phys. Rev. **D50** (1994) 632.

[13] P.I. Krastev and S.T. Petcov, Nucl. Phys. **B449** (1995) 605; P.I. Krastev, S.T. Petcov and L. Qiuyu preprint IASSNS-AST 96/11 (hep-ph/9602333).

[14] E. Calabresu, N. Ferrari, G. Fiorentini and M. Lissia, Astropart. Phys. **4** (1995) 159; P.I. Krastev and S.T. Petcov, preprint IASSNS-AST 95/40 (hep-ph/9510367).

[15] Kamiokande Collaboration, K.S. Hirata et al., Phys. Lett. **B205** (1988) 416; *ibid.* **B280** (1992) 146.

[16] Kamiokande Collaboration, Y. Fukuda et al., Phys. Lett. **B335** (1994) 237.

[17] IMB Collaboration, D. Casper et al., Phys. Rev. Lett. **66** (1989) 2561; R. Becker-Szendy et al., Phys. Rev. **D46** (1989) 3720.

[18] NUSEX Collaboration, M. Aglietta et al., Europhys. Lett. **8** (1989) 611.

[19] Frejus Collaboration, Ch. Berger et al., Phys. Lett. **B227** (1989) 489; *ibid.* **B245** (1990) 305; K. Daum et al, Z. Phys. **C66** (1995) 417.

[20] Soudan 2 Collaboration, M. Goodman et al., Nucl. Phys. **B** (Proc. Suppl.) **38** (1995) 337.

[21] For earlier references, see T.K. Kuo and J. Pantaleone, Rev. Mod. Phys. **61** (1989) 937 and references therein.

[22] K. Hidaka, M. Honda and S. Midorikawa, Phys. Rev. Lett. **61** (1988) 1537; S. Midorikawa, M. Honda and K. Kasahara, Phys. Rev. **D44** (1991) R3379; J. Pantaleone, Phys. Rev. **D43** (1991) 641; *ibid.* **D49** (1994) 2152; X. Shi and D.N. Schramm, Phys. Lett. **B283** (1992) 305; T.K. Kuo and J. Pantaleone, Phys. Rev. **D47** (1993) 4059; A. Acker, A.B. Balantekin and F. Loreti, Phys. Rev. **D49** (1994) 328.

[23] G.L. Fogli, E. Lisi, D. Montanino, Phys. Rev. **D49** (1994) 3626; G. Conforto, Nucl. Phys. **B** (Proc. Suppl.) **38** (1995) 308; C. Giunti, C.W. Kim and J.D. Kim, Phys. Lett. **B352** (1995) 357; H. Minakata, Phys. Lett. **B356** (1995) 61; Phys. Rev. **D52** (1995) 6630; S.M. Bilenky, A. Bottino, C. Giunti, C.W. Kim, Phys. Lett. **B356** (1995) 273; preprint DFTT 2/96 (hep-ph/9602216); S.M. Bilenky, C. Giunti, C.W. Kim and S.T. Petcov, preprint DFTT 18/96 (hep-ph/9604364); K.S. Babu, J.C. Pati and F. Wilczek, Phys. Lett. **B359** (1995) 351, **B364** (1995) 251 (E); M. Narayan, M.V.N. Murthy, G. Rajasekaran, S. Uma Sankar, Phys. Rev. **D53** (1996) 2809; G.L. Fogli, E. Lisi and G. Scioscia, Phys. Rev. **D52** (1995) 5334; S. Goswami, K. Kar and A. Raychaudhuri, preprint CUPP-95-3 (hep-ph/9505395).

[24] S.M. Bilenky, C. Giunti, C.W. Kim, preprint DFTT-30-95 (hep-ph/9505301).

[25] O. Yasuda, preprint TMUP-HEL-9603 (hep-ph/9602342).

[26] G.L. Fogli, E. Lisi and D. Montanino, preprint IASSNS-AST-96-21 (hep-ph/9605273).

[27] O. Yasuda and H. Minakata, preprint TMUP-HEL-9604 (hep-ph/9602386).

[28] C.Y. Cardall and G.M. Fuller, Phys. Rev. **D53** (1996) 4421.

[29] J.T. Peltoniemi and J.W.F. Valle, Nucl. Phys. **B406** (1993) 409.

[30] D.O. Caldwell and R.N. Mohapatra, Phys. Rev. **D48** (1993) 3259; Phys. Rev. **D50** (1994) 3477.

[31] J.R. Primack, J. Holtzman, A. Klypin and D. Caldwell, Phys. Rev. Lett. **74** (1995) 2160.

[32] J.J Gomez-Cadenas and M.C. Gonzalez-Garcia, preprint CERN-TH/95-80 (hep-ph/9504246).

[33] S. Goswami, preprint CUPP-95/4 (hep-ph/9507212).

[34] J.T. Peltoniemi, hep-ph/9506228.

[35] Y.-Z. Qian, G.M. Fuller, G.J. Mathews, R.W. Mayle, J.R. Wilson, Phys. Rev. Lett. **71** (1993) 1965.

[36] K. Kainulainen, J. Maalampi, and J.T. Peltoniemi, Nucl. Phys. **B358** (1991) 435.

[37] S.M. Bilenky, J. Hosek and S.T. Petcov, Phys. Lett. **94B** (1980) 495.

[38] J. Schechter and J.W.F. Valle, Phys. Rev. **D22** (1980) 2227.

[39] M. Doi, T. Kotani, H. Nishiura, K. Okuda and E. Takasugi, Phys. Lett. **102B** (1981) 323.

[40] S.M. Bilenky and S.T. Petcov, Rev. Mod. Phys. **59** (1987) 671; **61** (1989) 169 (E).

[41] M. Fukugita and T. Yanagida, *PHYSICS AND ASTROPHYSICS OF NEUTRINOS*, Springer-Verlag, 1994, eds. M. Fukugita and A. Suzuki, p1.

[42] C.-S. Lim, Proc. of the BNL Neutrino Workshop on Opportunities for Neutrino Physics at BNL, Upton, N.Y., February 5-7, 1987, ed. by M. J. Murtagh, p111.

[43] A. Yu. Smirnov, Proc. of the Int Symposium on Neutrino Astrophysics, Takayama/Kamioka 19 - 22 October 1992, ed. by Y. Suzuki and K. Nakamura, p.105.

[44] CDHSW Collaboration, F. Dydak et al., Phys. Lett. **134B** (1984) 281.

[45] R. Barbieri and A. Dolgov, Phys. Lett. **B237** (1990) 440; Nucl. Phys. **B349** (1991) 743.

[46] K. Kainulainen, Phys. Lett. **B244** (1990) 191.

[47] K. Enqvist, K. Kainulainen and M. Thomson, Nucl. Phys. **B373** (1992) 498; X. Shi, D.N. Schramm and B.D. Fields, Phys. Rev. **D48** (1993) 2563.

[48] C.J. Copi, D.N. Schramm, M.S. Turner, Phys. Rev. Lett. **75** (1995) 3981.

[49] N. Hata, R.J. Scherrer, G. Steigman, D. Thomas, T.P. Walker, S. Bludman, P. Langacker, Phys. Rev. Lett. **75** (1995) 3977.

[50] M.K. Moe, Nucl. Phys. **B** (Proc. Suppl.) **38** (1995) 36.

[51] A. Balysh et al., Proc. of the Int. Conference on High Energy Physics (ICHEP), 27th, Glasgow, Scotland, Eds. P.J. Bussey and I.G. Knowles, IOP, 1995, p.939 (hep-ex/9502007).

[52] R. Schaefer, Q. Shafi and F.W. Stecker, Astrophys. J. **347** (1989) 575; J.A. Holtzman and J.R. Primack, Astrophys. J. **405** (1993) 428; A. Klypin, J. Holtzman, J. Primack and E. Regos, Astrophys. J. **416** (1993) 1; Y.P. Jing, H.J. Mo, G. Borner and L.Z. Fang, Astron. Astrophys. **284** (1994) 703; K.S. Babu, R.K. Schaefer and Q. Shafi, Phys. Rev. **D53** (1996) 606.

[53] P.J. Kernan and S. Sarkar, preprint CWRU-P3-96 (astro-ph/9603045).

[54] C.Y. Cardall and G.M. Fuller, astro-ph/9603105.

[55] S.M. Bilenky and C. Giunti, Phys. Lett. **B320** (1994) 323; Z. Phys. **C68** (1995) 495.